

Complex Numbers Exercises

1. write in the form $x + iy$:

(a). $\frac{1}{2 + 2i}$

(b). $\frac{1}{i^3}$

(c). $i(1 + i)(1 - i)^2$

2. Write in the polar and the exponential polar form:

(a). $\frac{1}{2 + 2i}$

(b). $-1 + i\sqrt{3}$

(c). $\sqrt{1 + i}$

3. Give all roots (solutions) of $z^2 + z + 1 = 0$.

4. Split into factors: $z^2 + 1$.

5. Multiplying a complex z by i is the equivalent of rotating z in the complex plane by $\pi/2$.

(a). Verify this for $z = 2 + 2i$

(b). Verify this for $z = 4 - 3i$

(c). Show that $zi \perp z$ for all complex z .

6. Calculate $\text{Im}((i + 1)^8 z^2)$ for $z = x + iy$.

7. Find an expression for $\sin(3\theta)$ in terms of $\sin(\theta)$, $\cos(\theta)$.

8.(advanced) Solve $z^4 + 16=0$ for complex z , then use your answer to factor $z^4 + 16$ into two factors with real coefficients.